How to simulate simple economic models in *R*

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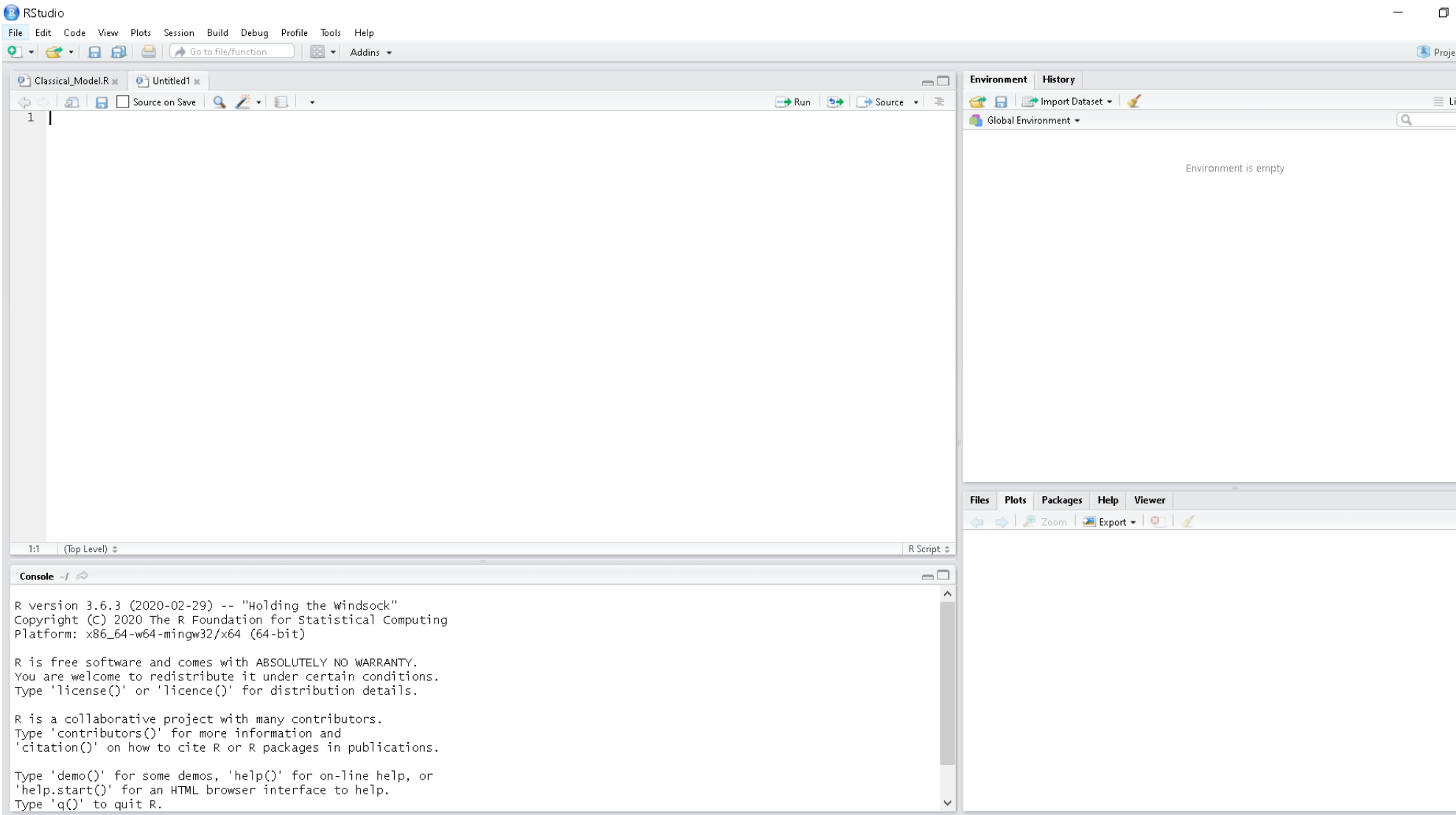
What is *R*?

- a programming language and free software environment for statistical computing and graphics (see here)
- popular across many sciences for statistical and numerical analyses
- unlike many other packages (e.g. *EViews*, *MATLAB*, *Mathematica* and *Stata*), **R is free**
- its functionality continuously grows thanks to numerous user-written libraries and packages
- *RStudio* provides an interface for *R*, which facilitates its use

How to get *R* running on your machine

- download and install both R and RStudio
- Open *R-Studio*
- you can now use *R*


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Go to file/function
Addins
Classical_Model.R
Untitled1
Source on Save
Run
Source
1
1:1
(Top Level)
R Script
Console ~/
R version 3.6.3 (2020-02-29) -- "Holding the Windsock"
Copyright (C) 2020 The R Foundation for Statistical Computing
Platform: x86_64-w64-mingw32/x64 (64-bit)
R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.
R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.
Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
Environment
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Environment is empty
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The main components of *RStudio*

- the top-left window is the script in which you type and save your code
- the top-right window displays objects and data you create
- the bottom-left windows is the console in which you can enter code that will be executed directly; it also displays some output
- the bottom-right window displays various things, most importantly plots that you create and help files

Numerical simulations in *R*

- numerical simulation of simple economic models doesn't require deep knowledge of *R*
- we will almost exclusively use the built-in functions of *Base R* (see here for a list of key commands)
- the key tool we will use to solve models is the *for-loop* (see below)

Numerical simulations of economic models: some general remarks (1/2)

- every economic model is essentially a system of N equations
- these N equations determine an equivalent number of N endogenous variables
- economic models can be:
  - *static*: time plays no role and all endogenous variables are determined simultaneously
  - *dynamic*: time matters and the endogenous variables change gradually over time
- the endogenous variables are typically interrelated: e.g. x determines y, but y also determines x
- these interrelationships can be:
  - *simultaneous*: x and y determine each other within the same period
  - *recursive*: x affects y only in $t+1$ and/or vice versa (only possible in dynamic models)

Numerical simulations of economic models: some general remarks (2/2)

- many simple economic models can be solved analytically without a computer
- numerical simulation can be useful:
  - if a model has more than 3 dimensions ($N > 3$) (then it's tedious to compute analytical solutions)
  - if a model has nonlinearities that preclude the computation of analytical solutions
  - to examine the dynamic adjustment of endogenous variables in dynamic models
- however, numerical solution requires the choice of a (possibly arbitrary) set of numerical values for the models' parameters
- it is thus less general than analytical solution

Numerical simulations of economic models: how (1/2)

- if a (dynamic) model exclusively contains recursive relationships, it can be solved iteratively by sequentially updating the endogenous variables from initial conditions
- by contrast, if interrelationships are simultaneous, solving the system for the endogenous variables is less trivial (e.g. finding the solution for x requires the solution of y, but the latter requires in turn the solution for x)
- one approach is to use linear algebra: cast the system in matrix form ($b = Ax$) and let the computer find $x^* = A^{-1}b$ through some algorithm (e.g. the Gauss-Seidel method)

Numerical simulations of economic models: how (2/2)

We will use an approach that is simpler and based on iteration:

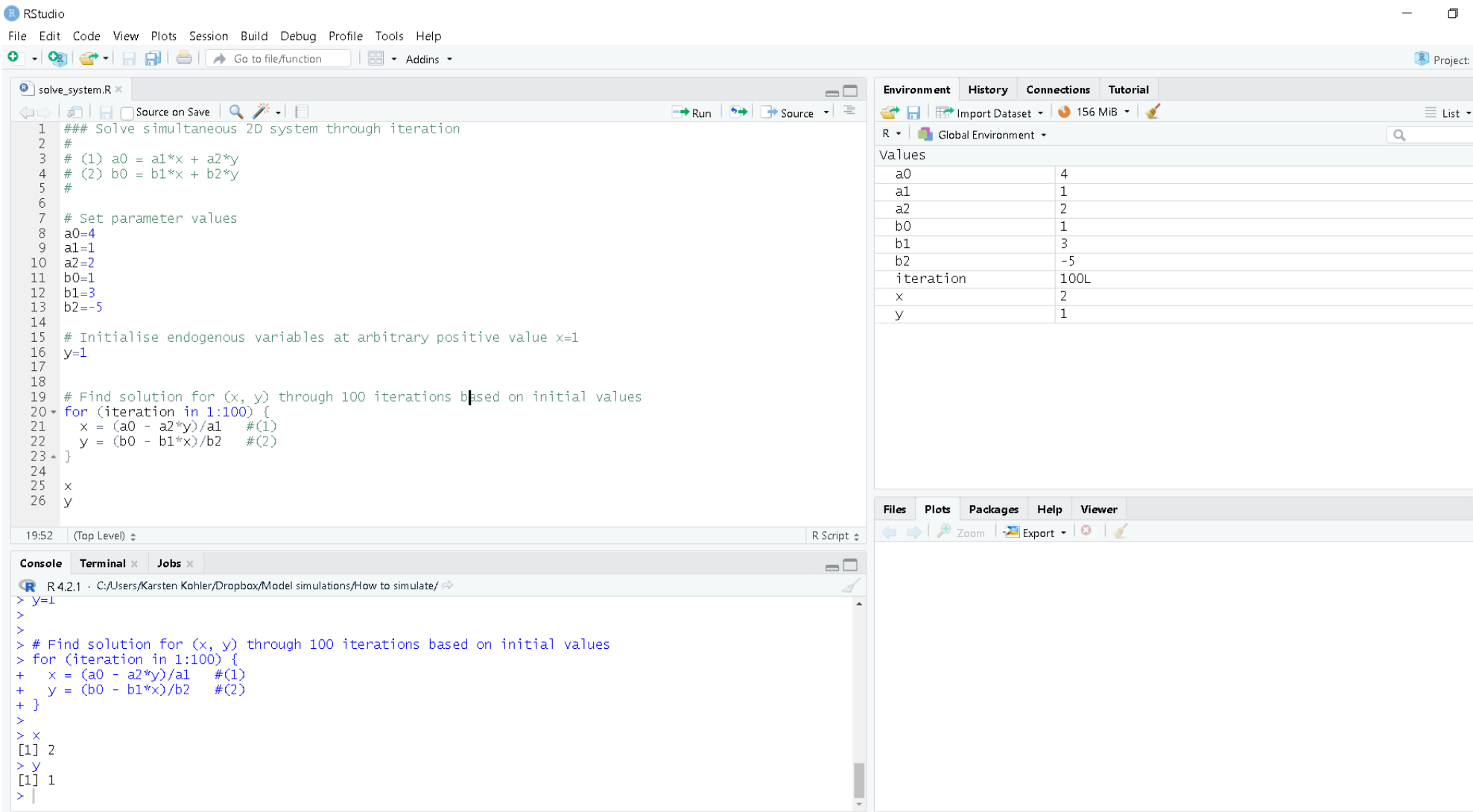
- choose (arbitrary) non-zero values for the (initial) endogenous variables
- then solve the equations many times through iteration using a for-loop
- in this way, the solution gets approximated successively

Numerical solution of a simultaneous system through iteration: example (1/3)

- consider the two-dimensional simultaneous system

$$a_0 = a_1 x + a_2 y \tag{1}$$
$$b_0 = b_1 x + b_2 y. \tag{2}$$

- suppose the parameters are given by $a_0 = 4, a_1 = 1, a_2 = 2, b_0 = 1, b_1 = 3, b_2 = -5$
- now you want to find the solution $(x^*, y^*) = (2, 1)$ through simulation


RStudio
File Edit Code View Plots Session Build Debug Profile Tools Help
Go to file/function
Addins
Project:
solve_system.R
Source on Save
Run
Source
Solve simultaneous 2D system through iteration
#
(1) a0 = a1*x + a2*y
(2) b0 = b1*x + b2*y
#
# Set parameter values
a0=4
a1=1
a2=2
b0=1
b1=3
b2=-5
# Initialise endogenous variables at arbitrary positive value x=1
y=1
# Find solution for (x, y) through 100 iterations based on initial values
for (iteration in 1:100) {
x = (a0 - a2*y)/a1 #(1)
y = (b0 - b1*x)/b2 #(2)
}
x
y
19:52
(Top Level)
R Script
Environment History Connections Tutorial
Import Dataset
156 MiB
List
R
Global Environment
Values
a0 4
a1 1
a2 2
b0 1
b1 3
b2 -5
iteration 100L
x 2
y 1
Files Plots Packages Help Viewer
Zoom
Export
Console Terminal Jobs
R 4.2.1 · C:/Users/Karsten Kohler/Dropbox/Model simulations/How to simulate/
> y=1
>
>
> # Find solution for (x, y) through 100 iterations based on initial values
> for (iteration in 1:100) {
+ x = (a0 - a2*y)/a1 #(1)
+ y = (b0 - b1*x)/b2 #(2)
+ }
>
> x
[1] 2
> y
[1] 1
>

Numerical solution of a simultaneous system through iteration: example (2/3)

- the code above solves the simultaneous system numerically through iteration
- you first set the parameter values
- then you initialise the endogenous variables at an arbitrary value (different from zero)
- write down the equations, solved for the endogenous variables, inside a for-loop
- then you solve the system iteratively through the for-loop
- the loop says: repeat the segment of code insights the curly brackets 100 times

Numerical solution of a simultaneous system through iteration: example (3/3)

- what happens is the following: in the first iteration, x and y are calculated based on the initial values and the parameter values
- in the second iteration, the values for x and y are then overwritten based on the results from the first iteration
- this process continues 100 times
- in this way, the correct solution $(x^*, y^*) = (2, 1)$ is successively approximated

Numerical solution of recursive system through iteration: example (1/2)

- now consider the two-dimensional recursive dynamic system

$$x_t = a_0 + a_1 x_{t-1} + a_2 y_{t-1} \tag{3}$$
$$y_t = b_0 + b_1 x_{t-1} + b_2 y_{t-1}. \tag{4}$$

- suppose the parameters are given by $a_0 = 1, a_1 = 0.3, a_2 = 0.6, b_0 = 1, b_1 = 0.3, b_2 = -0.2$
- now you want to find the solution $(x^*, y^*) \approx (2.73, 1.52)$ through simulation

RStudio

File Edit Code View Plots Session Build Debug Profile Tools Help

Go to file/function | Addins | Project:

solve_system.R

Source on Save | Run | Source

```
27
28  ## Solve recursive 2D system through iteration
29
30  #
31  # (1) x(t) = a0 + a1*x(t-1) + a2*y(t-1)
32  # (2) y(t) = b0 + b1*x(t-1) + b2*y(t-1)
33  #
34
35  # Set parameter values
36  a0=1
37  a1=0.3
38  a2=0.6
39  b0=1
40  b1=0.3
41  b2=-0.2
42
43  # Set number of periods
44  T=100
45
46  # Construct matrices in which values for different periods will be stored; initialise at 1
47  y=matrix(data=1, nrow=1, ncol=T)
48  x=matrix(data=1, nrow=1, ncol=T)
49
50  for (t in 2:T){
51      x[,t]=a0 + a1*x[,t-1] + a2*y[,t-1]
52      y[,t]=b0 + b1*x[,t-1] + b2*y[,t-1]
53  }
54
55  x[,T]
56  y[,T]
57
58  plot(x[2:T], type="l", ylab="X", xlab="time")
59
60
```

61:1 (Top Level) R Script

Console | Terminal | Jobs

R 4.2.1 · C:/Users/Karsten Kohler/Dropbox/Model simulations/How to simulate/

```
> x[,T]
[1] 2.727273
> y[,T]
[1] 1.515152
>
> plot(x[2:T], type="l", ylab="X", xlab="time")
>
```

Environment | History | Connections | Tutorial

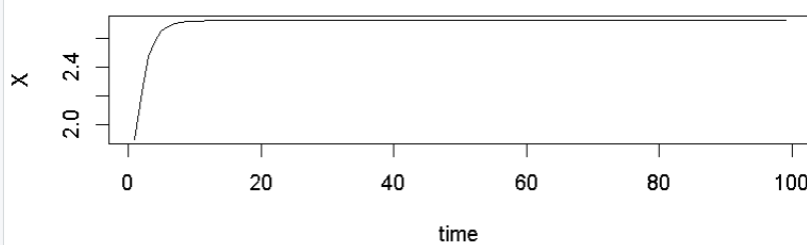
Import Dataset | 148 MiB | List

R | Global Environment

Data	
x	num [1, 1:100] 1 1.9 2.23 2.48 2.58 ...
y	num [1, 1:100] 1 1.1 1.35 1.4 1.46 ...
Values	
a0	1
a1	0.3
a2	0.6
b0	1
b1	0.3
b2	-0.2
t	100L
T	100

Files | Plots | Packages | Help | Viewer

Zoom | Export | Publish



Numerical solution of recursive system through iteration: example (2/2)

- this code solves the recursive system numerically through iteration
- we need to set a number of periods for which we want to simulate the model (here $T = 100$)
- we also create some matrices in which the solutions for each time step will be saved
- otherwise, the approach is the same: we iterate over the equations using a for-loop
- we can then plot the adjustment of the endogenous variables from the initial values to the solution

Getting started

- you can find a series of codes to simulate seminal economic models on my website
- all codes are accompanied by short notes that explain the models and also provide some analytical discussion
- feel free to get in touch if you have questions