A Post-Kaleckian Growth Model

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1 Overview

The post-Kaleckian growth model was developed by Bhaduri & Marglin (1990) and others to synthesise Marxian and Keynesian ideas about the effects of income distribution on economic growth.[1] According to the Marxian view, capital accumulation is driven by profits. By contrast, Michal Kalecki and post-Keynesians such as Nicholas Kaldor argued that a redistribution of income towards profit-earners is likely to reduce consumption, as workers tend to have a higher marginal propensity to consume than capital owners. The post-Kaleckian growth model integrates these two mechanisms in a Keynesian framework in which aggregate demand and growth are demand-determined. It allows for wage-led as well as profit-led demand and growth regimes. In a wage-led regime, a redistribution of income towards workers has expansionary effects on aggregate demand (and possibly growth) as the expansionary effect on consumption outweighs the negative effect on investment. In a profit-led regime, the effect is contradictory as investment falls by more than consumption rises. Whether a regime is wage- or profit-led depends on the relative size of the propensities to consume and the propensity to invest.

This is a model of long-run steady state growth. In the steady state, all endogenous variables grow at the same rate.[2] Changes in parameters or exogenous variables lead to an instantaneous adjustment of the model's variables, so that the model can be analysed like a static one. We consider a version of the model with linear functions based on Hein (2014, chap. 7.2.2).

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[*]I'm grateful to Eckhard Hein and Franz Prante for helpful discussions. All errors are mine.
[1]See Hein (2014, chap. 7) and Lavoie (2014, chap. 6) for detailed treatments.
[2]All variables are normalised by the capital stock and thus rendered stationary.

2 The Model

$$r = h\frac{u}{v} \tag{1}$$

$$s = [s_W + (s_\Pi - s_W)h]\frac{u}{v}, \qquad\qquad 0 \geq s_W > s_\Pi \geq 1 \tag{2}$$

$$c = \frac{u}{v} - s \tag{3}$$

$$g = g_0 + g_1 u + g_2 h, \qquad\qquad g_i > 0 \tag{4}$$

$$u = v(c + g) \tag{5}$$

where r, s, c, g, and u are the profit rate, the saving rate, the consumption rate, the investment rate, and the rate of capacity utilisation, respectively.

Equation (1) decomposes the profit rate into the product of the profit share h (total profits over total output), the rate of capacity utilisation (actual output over potential output), and the inverse of v (the capital-potential output ratio). Let Y be output, K be the capital stock, and Y^P be potential output, the decomposition can also be written as $r = \frac{\Pi}{K} = \frac{\Pi}{Y}\frac{Y}{Y^P}\frac{Y^P}{K}$. The profit share and the capital-potential output ratio are taken to be exogenous in this model. Note also that the wage share is given by $1 - h$. By equation (2), the economy-wide saving rate is given by the sum of saving out of wages ($s_W(1 - h)\frac{u}{v}$) and saving out of profits ($s_\Pi h\frac{u}{v}$). It is assumed that workers have a higher marginal propensity to consume than capital owners ($s_W > s_\Pi$). Equation (3) simply states that consumption is income not saved. According to equation (3), investment is determined by an autonomous component g_0 that may capture Keynesian 'animal spirits', by the rate of capacity utilisation, and by the profit share. While the rate of capacity utilisation is a signal of (future) demand, the profit share may stimulate investment as internal funds are typically the cheapest source of finance. Finally, equation (5) is an equilibrium condition that assumes that the rate of capacity utilisation adjusts to clear the goods market.

The key question addressed by this model is a how a change in the profit share affects the rate of capacity utilisation and the rate of growth.[3] As shown formally in the Appendix, there is no unambiguous answer to this question as the model encompasses different regimes. Three main regimes can be identified. First, a regime in which both the rate of utilisation and the rate of growth are negatively affected by an increase in the profit share. We will call this a wage-led demand and growth regime (WLD/WLG). Second, a regime in which the rate of utilisation is negatively affected and the rate of growth is positively affected by

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[3]Bhaduri & Marglin (1990) further discuss the effects on the profit rate.

an increase in the profit share. We will call this a wage-led demand regime and profit-led growth regime (WLD/PLG). Third, a regime in which both the rate of utilisation and the rate of growth are positively affected by an increase in the profit share. We will call this a profit-led demand and growth regime (PLD/PLG).

3   Simulation

Table 1 reports the parameterisation used in the simulation. We will consider three different parameterisations that represent the three regimes outlined above. For each of these regimes, there is a baseline scenario and a scenario in which the profit share (h) rises. This allows to assess the effects on the model's endogenous variables for the different regimes.

Table 1: Parameterisation

Scenario	v	s_W	s_Π	g_0	g_1	g_2	h
1a: baseline WLD/WLG	3	0.3	0.9	0.02	0.1	0.1	0.2
1b: rise in profit share (h)	3	0.3	0.9	0.02	0.1	0.1	0.3
2a: baseline WLD/PLG	3	0.3	0.9	0.02	0.08	0.1	0.2
2b: rise in profit share (h)	3	0.3	0.9	0.02	0.08	0.1	0.3
3a: baseline PLD/PLG	3	0.3	0.9	-0.01	0.1	0.1	0.2
3b: rise in profit share (h)	3	0.3	0.9	-0.01	0.1	0.1	0.3

Figures 1-4 depict the response of the model's key endogenous variables to changes in the profit share. In the first case of a wage-led demand and growth regime (WLD/WLG), investment is equally sensitive to a change in the rate of capacity utilisation (g_1) and a change in the profit share (g_2). A rise in the profit share reduces consumption, which reduces the rate of capacity utilisation and the rate of growth. This is despite a positive effect on the profit rate.[4]

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[4]If the negative effect on the rate of capacity utilisation was stronger, the profit rate could fall as well. See the Appendix for a formal derivation of the condition under which this may happen.

Figure 1: Rate of capacity utilisation
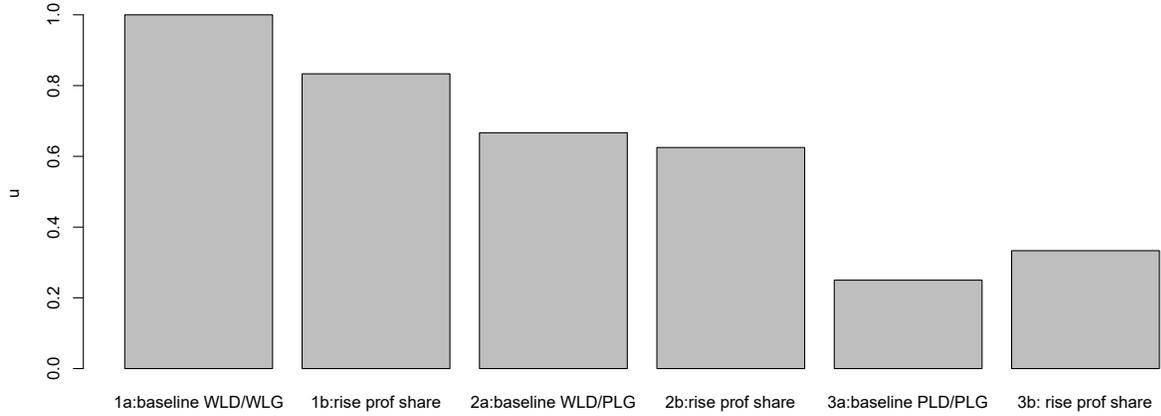
In the second case where the demand regime is wage-led but the growth regime is profit-led (WLD/PLG), investment is slightly less sensitive to a change in the rate of capacity utilisation compared to a change in the profit share. The rise in the profit share reduces consumption and the rate of utilisation, but the ultimate effect on investment is positive because investment reacts more strongly to the rise in the profit share than to the fall in demand.

Figure 2: Rate of growth
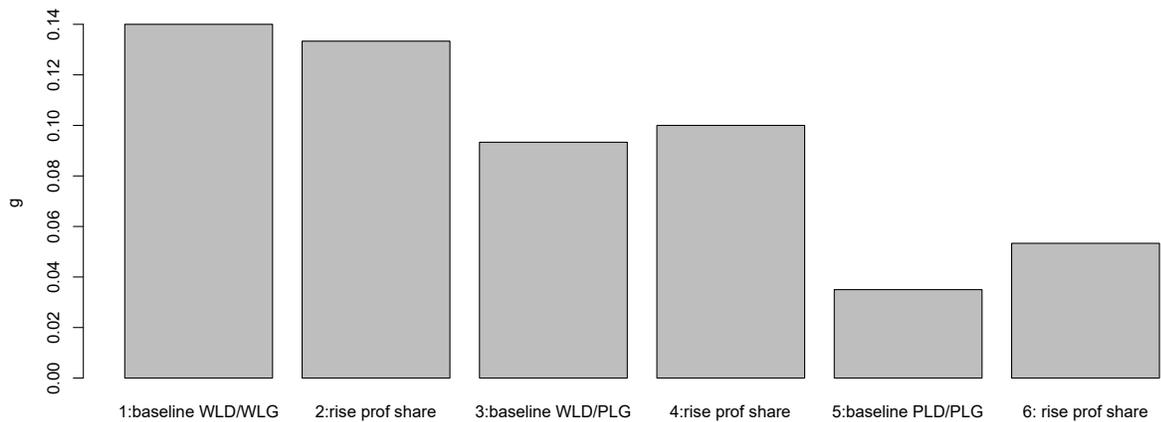
Finally, in the third case where the demand regime and the growth regime are profit-led, investment is again equally sensitive to a change in the rate of capacity utilisation and to

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a change in the profit share, but now animal spirits are negative. A rise in the profit share now has strong positive effects on investment, which raises the rate of capacity utilisation and consumption.
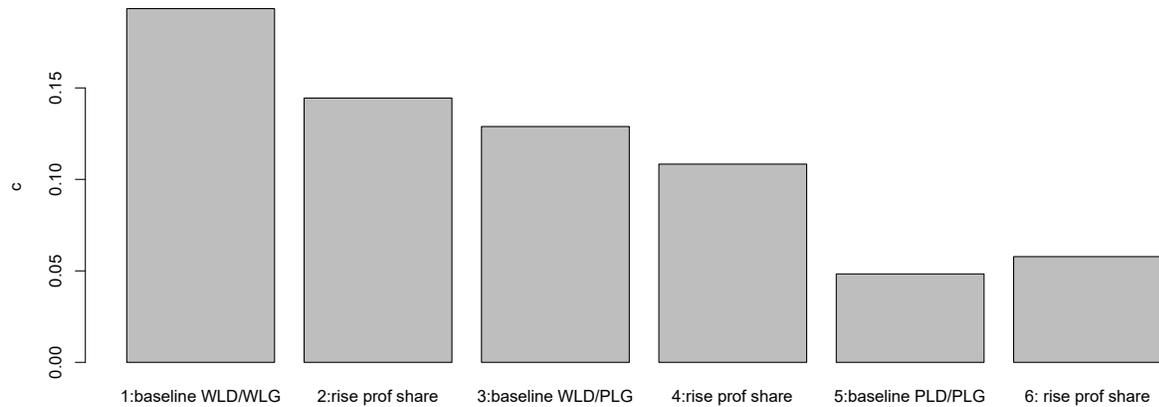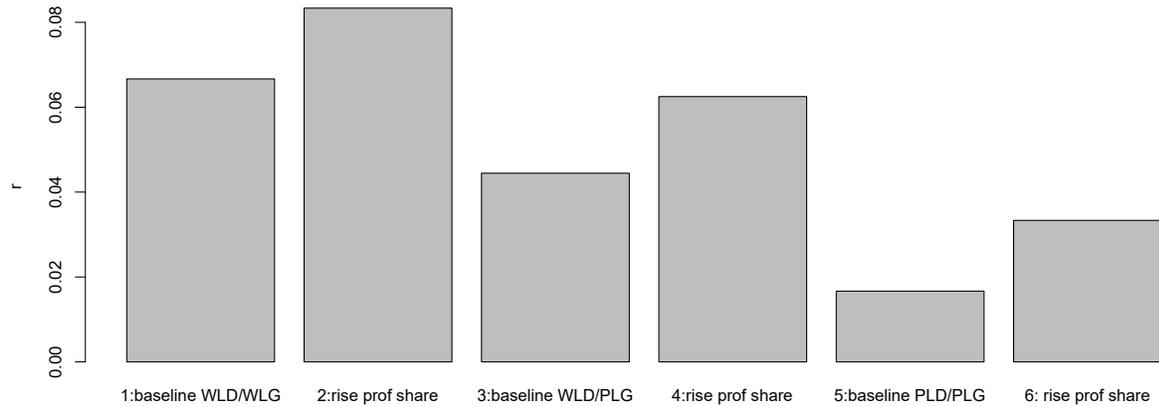
Figure 3: Rate of consumption


Figure 4: Rate of profit

4   Directed graph

Another perspective on the model's properties is provided by its directed graph. A directed graph consists of a set of nodes that represent the variables of the model. Nodes are connected

by directed edges. An edge directed from a node x_1 to node x_2 indicates a causal impact of x_1 on x_2.

Figure 5: Directed graph of post-Kaleckian growth model
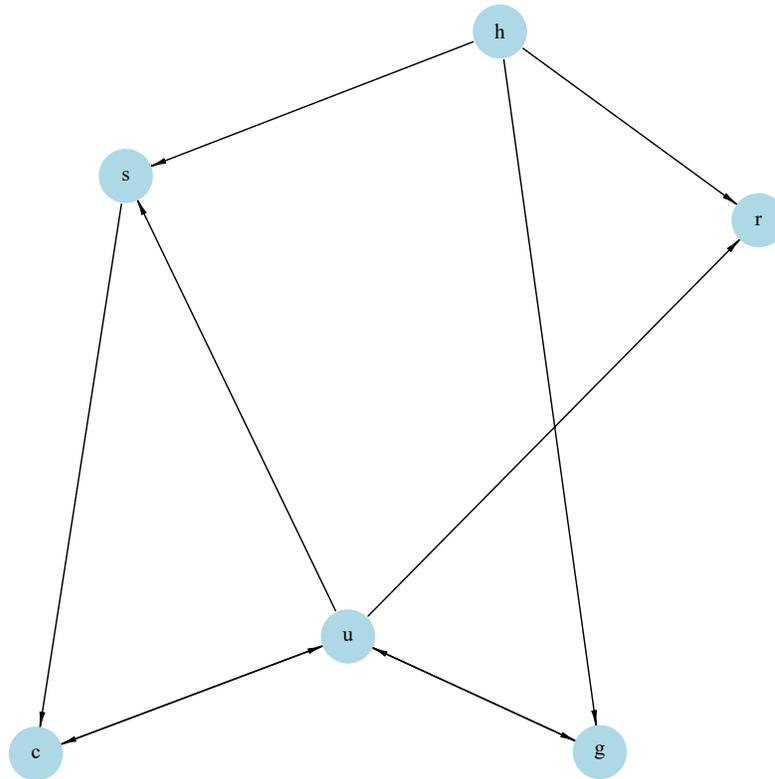
In Figure 5, it can be seen that the profit share (h) is the key exogenous variable of the model.[5] Consumption (c), saving (s), investment (g), and the rate of utilisation (u) form a closed loop (or cycle) within the system. The profit share affects both saving and investment, which in turn affect consumption and the rate of capacity utilisation. The profit rate is a residual variable (also called a 'sink') in this model.

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[5]Other important exogenous variables or parameters that may shift but are not depicted here are animal spirits (g_0) or the saving propensities (s_W, s_Π). See Hein (2014, chap. 7.2.2) for a detailed discussion of their effects.

Appendix

A   Analytical solution

To find the equilibrium solutions, substitute (2)-(4) into (5) and solve for u:

$$u^* = \frac{g_0 + g_2 h}{[s_W + (s_\Pi - s_W)h]v^{-1} - g_1}.$$

(6)

The equilibrium solution for u can then be substituted into (4) to find:

$$g^* = \frac{(g_0 + g_2 h)[s_W + (s_\Pi - s_W)h]v^{-1}}{[s_W + (s_\Pi - s_W)h]v^{-1} - g_1}.$$

(7)

The Keynesian stability condition requires $[s_W + (s_\Pi - s_W)h]v^{-1} - g_1 > 0$, i.e. saving need to react more strongly to income than investment.

The equilibrium solution for r can be found by substituting u^* into (1):

$$r^* = \frac{h(g_0 + g_2 h)}{[s_W + (s_\Pi - s_W)h] - vg_1}.$$

(8)

To assess whether the demand regime is wage- or profit-led, take the derivative of u^* with respect to h:

$$\frac{\partial u^*}{\partial h} = \frac{\frac{s_W}{v}(g_0 + g_2) - (g_0 \frac{s_\Pi}{v} + g_1 g_2)}{[[s_W + (s_\Pi - s_W)h]v^{-1} - g_1]^2}.$$

(9)

It can be seen that, e.g., a higher propensity to save out of wages or negative animal spirits make the regime more likely to be profit-led.

By the same token, the sign of the derivative of g^* with respect to h determines whether the growth regime is wage- or profit-led:

$$\frac{\partial g^*}{\partial h} = g_1 \frac{\partial u^*}{\partial h} + g_2.$$

(10)

It can be seen that, e.g., a higher sensitivity of investment with respect to the profit share makes the regime more likely to be profit-led.

Finally, the effect on the profit rate will depend on the sign of the derivative:

$$\frac{\partial r^*}{\partial h} = \frac{u^*}{v} + \frac{h}{v}\frac{\partial u^*}{\partial h}, \tag{11}$$

which is likely to be positive but can become negative if the demand regime is strongly wage-led.

B Construction of directed graph

The directed graph can be derived from the model's Jacobian matrix.[6] Let $\mathbf{x}$ be the vector containing the model's variables.[7] The system of equations representing the model can be written as $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. The Jacobian matrix is then given by $\mathbf{J} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$.

Next, construct an 'auxiliary' Jacobian matrix $\mathbf{M}$ in which the non-zero elements of the Jacobian are replaced by ones, whereas zero elements remain unchanged, i.e.

$$M_{ij} = \begin{cases} 1 & \text{if } J_{ij} \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Finally, taking the transpose of the 'auxiliary' Jacobian matrix yields the adjacency matrix ($\mathbf{M^T} = \mathbf{A}$), which is a binary matrix whose elements (A_{ji}) indicate whether there is a directed edge from a node x_j to node x_i. From the adjacency matrix, the directed graph is constructed.

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[6]See Fennell et al. (2015) for a neat exposition.
[7]Exogenous variables that are supposed to appear in the directed graph can readily be added to the Jacobian by an appropriate extension of rows and columns.

References

Bhaduri, A. & Marglin, S. (1990), 'Unemployment and the real wage: The economic basis for contesting political ideologies', *Cambridge Journal of Economics* **14**(4), 375–393.

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